AIAA 82-4076

An Adaptive Control Approach to Sensor Failure Detection and Isolation

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The present analysis introduces a software algorithm which monitors the operational status of the plant sensors. It introduces criteria by which a failed sensor is detected and isolated. The information obtained from the failed sensor will be disregarded and the system will continue to operate according to information obtained from the remaining unfailed sensors. The practicality and ease of implementation of the present technique is demonstrated by considering an aircraft equipped with a fly-by-wire control system performing a coordinated turn, where the roll rate gyro has become in the failure mode during the execution of the turn. Computer simulation has shown that the present technique has detected and isolated the faulty roll rate gyro and continued controlling the aircraft during its maneuver with insignificant effect on its performance quality.

Introduction

MEASUREMENTS play an essential role in digital control systems. Reliable estimates of the states of the controlled plant require the construction of a stable estimator and the availability of correct observation information. Sensor reliability and redundancy have been recognized as the corner stone for designing a digital control system. However, for plants operating in uncertain environment, the need for a technique that monitors the sensor operating mode and isolates the faulty one becomes of a paramount importance. In general, the availability of a software for sensor failure detection and isolation enhances the digital control system performance and reliability. Realizing this fact, the present method has been devised. A new technique for sensor failure detection and isolation for stable plants is presented. A measurement propagation equation is derived. Such equation is then cast in an interconnected subsystems form. An ideal observation vector is identified and equations for the observation error propagation (the difference between the ideal and the measured) are obtained. Owing to the coupling between the information obtained from different sensors, the sensor error propagation is dealt with as a linear-quadratic-Gaussian (LQG) optimal control problem. Such approach enables one to single out the ith sensor error propagation from the remaining ones. An ith sensor failure criterion is then established. To be able to implement the obtained criteria a reduced order observer is constructed. A matrix row switching and cycling algebra is introduced. Such algebra is used to build up the failed sensor isolation algorithm.

Analysis

Considering a stable plant which could be modeled by the discrete-time stochastic process

$$x_{k+1}=\Phi_k x_k + L_k u_k + w_k \qquad w_k \sim N(0,Q_k) \tag{1a}$$

$$z_k = H_k x_k + v_k \qquad v_k \sim N(0,R_k) \tag{1b}$$

where x is an n-dimensional state vector, u is an m-dimensional control vector, and z is a p-dimensional observation vector. It has been shown[1,3] that the state vector could be decomposed into an observable subvector of state, and a nonobservable subvector of state, i.e.,

$$x_k^T = [\underbrace{\omega_k^T}_{p} \,\vdots\, \underbrace{\nu_k^T}_{(n-p)}] \tag{2}$$

In the present analysis the observable subvector of state is defined as that one whose elements are directly related to the measurement vector. On the other hand, there is no direct relation between the measurement vector and the nonobservable subvector of state. In fact, the above partitioning is realized for a wide spectrum of aerospace systems. Based on the above concept, the state variables may be arranged such that the observation matrix may be partitioned into the form

$$H_k = [\underbrace{N_k}_{p} \,\vdots\, \underbrace{0}_{(n-p)}\,]\}^p \tag{3}$$

The type of sensors and sensor locations may be chosen to satisfy the above partitioning concept. Moreover, the additional requirement that matrix N be nonsingular could be easily realized. From Eqs. (2) and (3), Eq. (1b) may be written in the form

$$z_k = N_k \omega_k + v_k \tag{4}$$

Introducing the matrices Ψ and Γ such that

$$H_{k+1}\Phi_k = [\underbrace{\Psi_k}_{p} \,\vdots\, \underbrace{\Gamma_k}_{(n-p)}\,]\}^p \tag{5}$$

An observation propagation equation may be obtained from Eqs. (1a) and (2-4) as

$$z_{k+1} = \Psi_k N_k^{-1} z_k + \Gamma_k \nu_k + H_{k+1} L_k u_k + \epsilon_k \tag{6}$$

where

$$\epsilon_k = H_{k+1} w_k + v_{k+1} - \Psi_k N_k^{-1} v_k \tag{7}$$

Equation (6) represents the observation dynamics and vector ϵ is the observation dynamics noise. It is seen from Eq. (7) that the observation dynamics noise is Gaussian. If the sensors are functioning perfectly, the obtained measurements at different sample times should follow Eq. (6).

Accordingly Eq. (6) may be regarded as a reference trajectory in the observation space. Such trajectory will be computed and compared with the actual obtained

Received April 13, 1981; revision received Sept. 21, 1981.

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measurements. The computed (ideal) observation will be identified by superscript c, i.e., Eq. (6) may be rewritten as

$$z^c_{k+1} = \Psi^*_k z^c_k + \Gamma_k \nu_k + L^*_k u_k + \epsilon_k \tag{8}$$

where

$$\Psi^*_k = \Psi_k N^{-1}_k \qquad L^*_k = H_{k+1} L_k \tag{9}$$

Regarding all the sensors as an interconnected system with each sensor representing a subsystem, Eq. (8) may be decomposed as

$$\begin{aligned} z^{c(i)}_{k+1} &= \beta_k z^{c(i)}_k + b^T_k z^{c(r)}_k + d^T_k \nu_k + \alpha^T_k u_k + \epsilon^{(i)}_k \\ z^{c(r)}_{k+1} &= A_k z^{c(r)}_k + \eta_k z^{c(i)}_k + E_k \nu_k + \Theta_k u_k + \epsilon^{(r)}_k \end{aligned} \tag{10}$$

where the superscript i indicates the ith sensor and superscript r indicates the remaining sensors grouped together, and

$$\Gamma_k = \underbrace{\begin{bmatrix} d^T_k \\ \hline E_k \end{bmatrix}}_{(n-p)} \begin{matrix} \}\, 1 \\ \}\, (p-1) \end{matrix} \qquad L^*_k = \underbrace{\begin{bmatrix} \alpha^T_k \\ \hline \Theta_k \end{bmatrix}}_{m} \begin{matrix} \}\, 1 \\ \}\, (p-1) \end{matrix}$$

$$\Psi^*_k = \begin{bmatrix} \underbrace{\beta_k}_{1} & \underbrace{b^T_k}_{(p-1)} \\ \hline \eta_k & A_k \end{bmatrix} \begin{matrix} \}\, 1 \\ \}\, (p-1) \end{matrix} \qquad \epsilon^T_k = [\underbrace{\epsilon^{(i)}_k}_{1} \;\vdots\; \underbrace{\epsilon^{(r)\,T}_k}_{(p-1)}]$$

$$z^{cT}_k = [\underbrace{z^{c(i)}_k}_{1} \mid \underbrace{z^{c(r)\,T}_k}_{(p-1)}] \tag{11}$$

Denoting the actual measurement vector by superscript m and the difference between the calculated and measured observations by $\Delta z = z^c - z^m$, the following equations may be obtained from Eqs. (10):

$$\begin{aligned} \Delta z^{(i)}_{k+1} = \beta_k \Delta z^{(i)}_k + b^T_k \Delta z^{(r)}_k + [&-z^{m(i)}_{k+1} \\ &+ \beta_k z^{m(i)}_k + b^T_k z^{m(r)}_k + d^T_k \nu_k + \alpha^T_k u_k + \epsilon^{(i)}_k] \end{aligned} \tag{12a}$$

$$\begin{aligned} \Delta z^{(r)}_{k+1} = A_k \Delta z^{(r)}_k + \eta_k \Delta z^{(i)}_k + [&-z^{m(r)}_{k+1} \\ &+ A_k z^{m(r)}_k + \eta_k z^{m(i)}_k + E_k \nu_k + \Theta_k u_k + \epsilon^{(r)}_k] \end{aligned} \tag{12b}$$

For perfectly operating sensors the expressions between brackets in the above equations vanish and Eqs. (12) reduce to

$$\Delta z^{(i)}_{k+1} = \beta_k \Delta z^{(i)}_k + b^T_k \Delta z^{(r)}_k \tag{13a}$$

$$\Delta z^{(r)}_{k+1} = A_k \Delta z^{(r)}_k + \eta_k \Delta z^{(i)}_k \tag{13b}$$

Equations (13) are coupled error equations representing the propagation of the deviations from the ideal conditions of the information obtained from the sensors. It is seen that information error due to one sensor malfunction generates errors in the processing of signals obtained from other sensors. For perfectly operating sensors, $\Delta z^{(i)}$ as well as $\Delta z^{(r)}$ should be identically zero. However, owing to practical considerations such differences are not exactly zero. The magnitude of such deviations is a measure of the sensors' operating conditions. The acceptable domain of sensor operation is bounded by the minimum information errors $\Delta z^{(i)}$ and $\Delta z^{(r)}$. This is equivalent to minimizing $\Delta z^{(r)}$ and $\Delta z^{(i)}$ subject to the difference constraint given by Eq. (13a). Besides, employment of the optimal control approach decouples the sensor deviations $\Delta z^{(i)}$ and $\Delta z^{(r)}$. The proper performance index for such requirement is

$$J^{(i)} = \frac{1}{2} E\left\{ \sum_{k=0}^{N-1} [q^*_k \Delta z^{(i)\,2}_k + \Delta z^{(r)\,T}_k R^*_k \Delta z^{(r)}_k] \right\} \tag{14}$$

The difference vector $\Delta z^{(r)}$ that minimizes the above quadratic performance index may be obtained as

$$\Delta z^{(r)}_k = -R^{*\,-1}_k b_k s_{k+1} \Delta z^{(i)}_{k+1} = -C^{r \to i}_{k/k+1} \Delta z^{(i)}_{k+1} \tag{15}$$

The scalar variable s is calculated by means of the equation

$$s_k = q^*_k + \beta^2_k s_{k+1} / (1 + b^T_k R^{*\,-1}_k b_k s_{k+1}) \qquad s_N = 0 \tag{16}$$

where q^* is a weighting parameter and R^* is a weighting matrix. Substitution of Eq. (15) into Eq. (13a) yields

$$\Delta z^{(i)}_{k+1} = [\beta_k / (1 + b^T_k C^{r \to i}_{k/k+1})] \Delta z^{(i)}_k \tag{17a}$$

Based on Eq. (17a) let us define the acceptable domain of the ith sensor operation by $\Delta y^{(i)}$, where

$$\Delta y^{(i)}_{k+1} = (1 + \xi) [\beta_k / (1 + b^T_k C^r_{k/k+1})] \Delta y^{(i)}_k \tag{17b}$$

ξ is a number that accounts for the computation round-off error. If the difference $\Delta z^{(i)}$ falls within such domain the ith sensor is considered in the operational mode. The above equation is initialized by assigning a proper value for $\Delta y^{(i)}_0$. The ith sensor failure detection criterion may thus be stated as, "The ith sensor will be considered in the failure mode if the following inequality is satisfied."

$$|\Delta z^{(i)}_{k+1}| > |\Delta y^{(i)}_{k+1}| \tag{18}$$

In order to be able to implement the above criterion $z^{c(i)}$ should be calculated by means of Eqs. (10), which involve the nonobservable subvector of state ν. An estimate of ν is then required to perform the needed computations. Following the approach of Ref. 2, a reduced order observer may be constructed in the following manner. Let us partition the plant state transition and the control sensitivity matrices as

$$\Phi_k = \begin{bmatrix} \underbrace{\Phi^{(1)}_k}_{p} & \underbrace{\Phi^{(2)}_k}_{(n-p)} \\ \hline \Phi^{(3)}_k & \Phi^{(4)}_k \end{bmatrix} \begin{matrix} \}\, p \\ \}\, (n-p) \end{matrix} \qquad L_k = \underbrace{\begin{bmatrix} L^{(1)}_k \\ \hline L^{(2)}_k \end{bmatrix}}_{m} \begin{matrix} \}\, p \\ \}\, (n-p) \end{matrix} \tag{19}$$

From Eqs. (1a), (2), (4), and (19) we may thus get

$$\nu_{k+1} = \Phi^{(4)}_k \nu_k + \Phi^{(3)}_k N^{-1}_k z_k + L^{(2)}_k u_k + (w_{2k} - N^{-1}_k v_k) \tag{20}$$

where

$$w^T_k = [\underbrace{w^T_{1k}}_{p} \mid \underbrace{w^T_{2k}}_{(n-p)}]$$

Based on the assumption that the plant noise and the measurement noise vectors are white Gaussian, Eq. (20) provides the reduced order observer

$$\hat{\nu}_{k+1} = \Phi^{(4)}_k \hat{\nu}_k + \Phi^{(3)}_k N^{-1}_k z_k + L^{(2)}_k u_k \tag{21}$$

Introducing the following partitioning

$$\Phi^{(3)}_k N^{-1}_k = [\underbrace{\mu_k}_{1} \mid \underbrace{M_k}_{(p-1)}] \}\, (n-p) \tag{22}$$

Based on the last of Eqs. (11) and Eq. (22) and identifying the observation vector appearing in Eq. (21) by z^c, Eq. (21) may be written in the form

$$\hat{v}_{k+1} = \Phi_k^{(4)} \hat{v}_k + \mu_k z_k^{c(i)} + M_k z_k^{c(r)} + L_k^{(2)} u_k \tag{23}$$

In the above equation the observation vector was considered to be the calculated one in order to construct the ideal observation trajectory. To reduce the computation time of the sensor failure detection algorithm, a dual failure detection criterion is established. That is, simultaneously with the checking of the ith sensor performance, the dual criterion will check the performance of the remaining sensors. Considering Eq. (13b), the difference scalar $\Delta z^{(i)}$ that minimizes the quadratic performance index

$$J^{(r)} = \frac{1}{2} E\left\{ \sum_{k=0}^{N-1} [\Delta z_k^{(r)T} Q_k^* \Delta z_k^{(r)} + r_k^* \Delta z_k^{(i)2}] \right\} \tag{24}$$

is obtained as

$$\Delta z_k^{(i)} = -r_k^{*-1} \eta_k^T P_{k+1} \Delta z_{k+1}^{(r)} = -C_{k/k+1}^{i-r} \Delta z_{k+1}^{(r)} \tag{25}$$

where matrix P is governed by the equation

$$P_k = Q_k^* + A_k^T P_{k+1} [I + (\eta_k \eta_k^T / r_k^*) P_{k+1}]^{-1} A_k \qquad P_N = 0 \tag{26}$$

In the above equations Q^* is a weighting matrix and r^* is a weighting parameter. Substitution of Eq. (25) into Eq. (13b) yields

$$\Delta z_{k+1}^{(r)} = [I + \eta_k C_{k/k+1}^{i-r}]^{-1} A_k \Delta z_k^{(r)} \tag{27a}$$

Similar to the ith sensor and based on Eq. (27a) an acceptable domain of all the remaining sensors operation is defined by $\Delta y^{(r)}$ where

$$\Delta y_{k+1}^{(r)} = (1+\xi) [I + \eta_k C_{k/k+1}^{i-r}]^{-1} A_k \Delta y_k^{(r)} \tag{27b}$$

If the difference $\Delta z^{(r)}$ falls within such domain, all the remaining sensors (other than the ith sensor) will be considered in the operational mode.

A failure detection criterion may thus be stated as, "One or more of the remaining sensors (other than the ith sensor) will be considered in the failure mode if the following inequality is satisfied."

$$\|\Delta z_{k+1}^{(r)}\| > \|\Delta y_{k+1}^{(r)}\| \tag{28}$$

where $\|\cdot\|$ indicates the norm and ξ is a number that accounts for computation round-off errors.

In order to be able to utilize the sensors failure detection criteria, the estimate of the nonobservable subvector of state should be used in Eqs. (10) rather than their actual values. Taking the expectation of both sides of Eqs. (10) yields

$$z_{k+1}^{c(i)} = \beta_k z_k^{c(i)} + b_k^T z_k^{c(r)} + d_k^T \hat{v}_k + \alpha_k^T u_k \tag{29a}$$

$$z_{k+1}^{c(r)} = A_k z_k^{c(r)} + \eta_k z_k^{c(i)} + E_k \hat{v}_k + \Theta_k u_k \tag{29b}$$

The sensors failure detection algorithm may thus be summarized in the following statements (noting that the gain matrices C^{r-i} and C^{i-r} are computed off-line and stored):

1) Starting with the initial conditions $\hat{v}_0$, $z_0^{c(i)} = z_0^{m(i)}$, $z_0^{c(r)} = z_0^{m(r)}$, $\Delta y_0^{(i)}$ and $\Delta y_0^{(r)}$, Eqs. (23) and (29) are computed simultaneously at each computation step.

2) At each sample time, based on the measured values $z^{m(i)}$ and $z^{m(r)}$, compute $\Delta z^{(i)}$ and $\Delta z^{(r)}$ and also compute $\Delta y^{(i)}$ and $\Delta y^{(r)}$.

3) For preassigned values of ξ, R^*, Q^*, r^*, and q^*, check inequalities (18) and (28).

4) If inequality (18) is satisfied, the ith sensor will be considered in the failure mode, otherwise it will be considered in the operating mode.

5) If inequality (28) is satisfied, this indicates that one or more of the remaining sensors (other than the ith sensor) has/have failed, otherwise all the remaining sensors are operative.

6) If inequality (18) is not satisfied and inequality (28) is satisfied, the sensors' designations should be switched such that the next sensor to the ith one is designated as the ith sensor.

7) The switching process should continue until all sensors have been checked.

The present technique is applicable for redundant as well as nonredundant systems. In fact, the presented analysis deals directly with nonredundant systems where the system states are classified as observable and nonobservable states. This was done because the number of measurements is less than the number of states. In such case, an observer for the nonobservable states should be constructed, Eq. (23). On the other hand, if redundant systems are dealt with, where the number of measurements exceeds the number of states, the measurement vector will be partitioned rather than the state vector as

$$z^T = [\underbrace{\zeta^T}_{n} \mid \underbrace{\zeta^{*T}}_{p-n}] \qquad (p>n) \tag{30}$$

The observation matrix is conveniently partitioned as

$$H = \underbrace{\begin{bmatrix} N \\ \hdashline N^* \end{bmatrix}}_{n} \begin{matrix} \}\, n \\ \}\, (p-n) \end{matrix} \tag{31}$$

such that N is a nonsingular matrix.

In this case Eqs. (5), (9), and (11) are modified to

$$H_{k+1}\Phi_k = \underbrace{\begin{bmatrix} \bar{\Psi}_k \\ \hdashline \bar{\Gamma}_k \end{bmatrix}}_{n} \begin{matrix} \}\, n \\ \}\, (p-n) \end{matrix} \qquad \bar{\Psi}_k^* = \bar{\Psi}_k N_k^{-1}$$

$$L_k^* = \underbrace{\begin{bmatrix} \bar{\alpha}_k^T \\ \hdashline \bar{\theta}_k \\ \hdashline \theta_k^* \end{bmatrix}}_{m} \begin{matrix} \}\, 1 \\ \}\, (n-1) \\ \}\, (p-n) \end{matrix} \qquad \bar{\Psi}^* = \begin{bmatrix} \bar{\beta}_k & \bar{b}_k^T \\ \hdashline \underbrace{\bar{\eta}_k}_{1} & \underbrace{\bar{A}_k}_{(n-1)} \end{bmatrix} \begin{matrix} \}\, 1 \\ \}\, (n-1) \end{matrix}$$

$$\epsilon_k^T = [\underbrace{\epsilon_k^{(i)}}_{1} \mid \underbrace{\epsilon_k^{(r)T}}_{(n-1)} \mid \underbrace{\epsilon_k^{*T}}_{(p-n)}] \qquad \zeta_k^{cT} = [\underbrace{\zeta_k^{c(i)}}_{1} \mid \underbrace{\zeta^{c(r)T}}_{n-1}] \tag{32}$$

Following similar analysis the ideal observation trajectory is obtained by

$$\zeta_{k+1}^{c(i)} = \bar{\beta}_k \zeta_k^{c(i)} + \bar{b}_k^T \zeta_k^{c(r)} + \bar{\alpha}_k^T u_k \tag{33a}$$

$$\zeta_{k+1}^{c(r)} = \bar{A}_k \zeta_k^{c(r)} + \bar{\eta}_k \zeta_k^{c(i)} + \bar{\theta}_k u_k \tag{33b}$$

For redundant systems, equations similar to Eqs. (15-18) and (25-28) are obtained with barred quantities instead of unbarred ones, $\Delta\zeta^{(i)}$ instead of $\Delta z^{(i)}$ and $\Delta\zeta^{(r)}$ instead of $\Delta z^{(r)}$.

If a sensor is found to be in the failure mode, it has to be isolated. An isolation algorithm is presented. It is based on the reduction of the dimension of the observation equation. Such reduction with the proper rearrangement of the state transition matrix and the control sensitivity matrix together with the subsequent derived matrices introduced in the analysis form the sensor isolation algorithm. The algebra needed for building up the sensor isolation algorithm is defined through the following definitions.

Fig. 1 Upward (left) and downward (right) circular shifting.

Definition 1. The rows of a square matrix D may be shifted in a circular manner by means of the following relations:

Upward Circular Shifting (UCS)

$$\overset{\blacktriangle}{\Phi} D = D_u$$

Downward Circular Shifting (DCS)

$$\underset{\blacktriangledown}{\Phi} D = D_d$$

where D is the basic square matrix, D_u is the rearranged matrix according to Fig. 1a, and D_d is the rearranged matrix according to Fig. 1b.

$\overset{\blacktriangle}{\Phi}$ is an upward circular shifting matrix (UCSM) which is basically an identity matrix with its first row shifted to the nth row, the $(n-1)$ row shifted to the $(n-2)$ row, and so forth.

$\underset{\blacktriangledown}{\Phi}$ is a downward circular shifting matrix (DCSM) which is basically an identity matrix with its first row shifted to the second row, the second row shifted to the third row, and so forth. The following examples explain the above shifting processes:

$$\underset{\overset{\blacktriangle}{\Phi}}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}} \underset{D}{\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix}} = \underset{D_u}{\begin{bmatrix} d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \\ d_{11} & d_{12} & d_{13} & d_{14} \end{bmatrix}}$$

$$\underset{\underset{\blacktriangledown}{\Phi}}{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \underset{D}{\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix}} = \underset{D_d}{\begin{bmatrix} d_{41} & d_{42} & d_{43} & d_{44} \\ d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \end{bmatrix}}$$

Definition 2. Any two rows of a matrix D (square or rectangular) may change places by means of the relation

$$\underset{(n\times n)}{[i \updownarrow j]} \underset{(n\times m)}{D} = \underset{(n\times m)}{D_r}$$

where D is the basic matrix and D_r is the rearranged one with the ith and jth rows interchanging places.

$[i \updownarrow j]$ is a row interchanging matrix (RIM) which is basically an identity matrix with the ith and jth rows interchanging places. For example,

$$\underset{2 \updownarrow 3}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{D}{\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix}} = \underset{D_r}{\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix}}$$

Definition 3. The last row of a $(p\times n)$ matrix F $(p<n)$ may be deleted by means of the relation

$$(p-1)\{[\underbrace{I}_{(p-1)} \mid \underbrace{0}_{1}] \underset{(p\times n)}{F} = \underset{[(p-1)\times n]}{F^*}$$

where I is a $(p-1)$ square identity matrix. For example,

$$\underset{[I \mid 0]}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}} \underset{F}{\begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \end{bmatrix}} = \underset{F^*}{\begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \end{bmatrix}}$$

The above three definitions constitute the basic algebra needed for building up the failed sensor isolation algorithm. If sensor No. 1 is found to be in the failure mode, such sensor is isolated and the present sensor failure detection technique is to be employed to check the remaining sensors through the following steps:

1) Replace the system transition matrix Φ by Φ^*, where

$$\Phi_k^* = \overset{\blacktriangle}{\Phi}\Phi_k$$

2) Replace the control sensitivity matrix L by L^+, where

$$L_k^+ = \overset{\blacktriangle}{\Phi} L_k$$

3) Replace the observation matrix H by H^-, where

$$H_k^- = \underset{(p-1)\times(p-1)\ \ (p-1)\times 1}{[I \mid 0]} \overset{\blacktriangle}{\Phi} H_k$$

4) Matrices N, Ψ, and Γ are replaced by $\bar{N}$, $\bar{\Psi}$, and $\bar{\Gamma}$, where

$$H_k^- = [\underbrace{\bar{N}_k}_{(p-1)} \mid \underbrace{0}_{(n-p+1)}]\}(p-1) \qquad H_{k+1}^- \Phi_k^* = [\underbrace{\bar{\Psi}_k}_{(p-1)} \mid \underbrace{\bar{\Gamma}_k}_{(n-p+1)}]\}(p-1)$$

5) Matrices Ψ^* and L^* are replaced by $\bar{\Psi}^*$ and $\bar{L}^*$, where

$$\bar{\Psi}_k^* = \bar{\Psi}_k \bar{N}_k^{-1} \qquad \bar{L}_k^* = H_{k+1}^- L_k^+$$

6) Replace the dimension p by $(p-1)$ in the partitioned matrices defined by Eqs. (11) and (19).

The failure detection and isolation of the controls sensors may proceed in the same manner as that done for the plant sensors. In such case, the controls actuator dynamics will be regarded as the plant equations. There are two ways to deal with this problem: 1) forming augmented plant and observation equations by adjoining the actuators dynamics and measurements to those of the plant, then using the present algorithm; and 2) using the present sensor failure detection and isolation algorithm simultaneously for the plant dynamics and observations and the control dynamics and observations. From the computational point of view, the second option is more efficient owing to the reduced order matrix Riccati equations involved instead of a full (augmented) order matrix Riccati equation that results by following the first option. When a control sensor fails it will be detected and isolated by the present method. Its effect on the plant performance (in the interval between sensor failure detection and convergence of the present estimator algorithm to reasonably accurate estimates of the plant control positions which might last for several sample intervals) will be that of an erroneous command input signal.

Application

Considering a typical VSTOL aircraft whose lateral dynamics may be represented by Eq. (1a), where

$$\Phi = \begin{bmatrix} 0.9949 & 0.0 & -0.1 & 0.0238 \\ -0.0737 & 0.08665 & 0.0369 & 0.001 \\ 0.001 & 0.0107 & 0.9668 & 0 \\ 0 & 0.1 & 0.0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0.004 & 0 \\ 0.127 & -2.031 \\ -0.206 & 0.133 \\ 0.004 & 0 \end{bmatrix}$$

$$x^T = [\beta p r \phi] \qquad u^T = [\delta_r \delta_a]$$

β is the angle of sideslip, p is the roll velocity, r is the yaw velocity, ϕ is the roll angle, δ_r is the rudder deflection angle, and δ_a is the ailerons deflection angle.

The observations are obtained from two single-axis rate gyros and a vertical gyro, i.e., in such case the observation matrix becomes

$$H = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The diagonal elements of the process noise covariance matrix are taken as (0.01, 0.008, 0.01, 0) and the diagonal elements of the measurement noise covariance matrix are (0.0001, 0.0001, 0.0003). In order to employ the present approach the state vector elements are rearranged as

$$x^T = [p r \phi \beta]$$

In such case matrices Φ, L, and H become

$$\Phi = \begin{bmatrix} 0.867 & 0.037 & 0 & -0.074 \\ 0.011 & 0.967 & 0 & 0.001 \\ 0.1 & 0 & 1.0 & 0 \\ 0 & -0.1 & 0.024 & 0.995 \end{bmatrix}$$

$$L = \begin{bmatrix} 0.127 & -2.031 \\ -0.206 & 0.133 \\ 0 & 0 \\ 0.004 & 0 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Based on the notation of the present analysis, the following are obtained:

$$N = I \qquad d = -0.074 \qquad E = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.866 & 0.037 & 0 \\ 0.011 & 0.967 & 0 \\ 0.1 & 0 & 1 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} -0.074 \\ 0.001 \\ 0 \end{bmatrix}$$

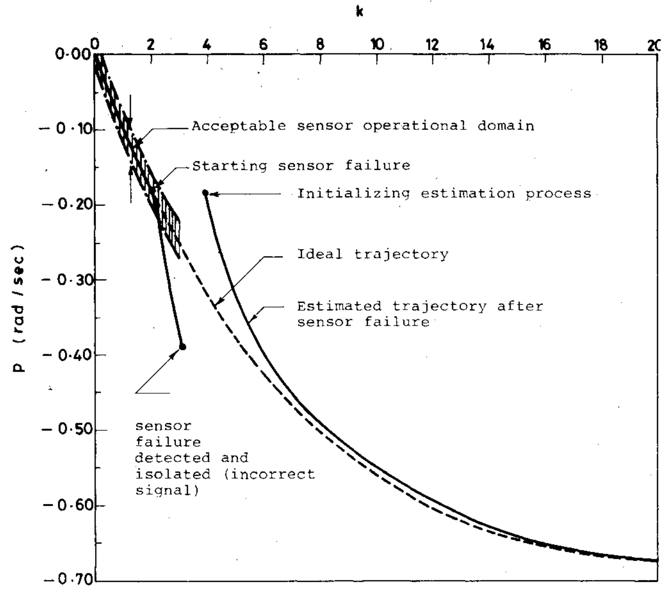

Fig. 2 The effect of roll rate gyro failure on estimated roll rate.

$$\alpha^T = [0.127 \quad -2.031] \qquad \beta = 0.866$$

$$\Theta = \begin{bmatrix} -0.206 & 0.133 \\ 0 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 0.967 & 0 \\ 0 & 1 \end{bmatrix}$$

$$b^T = [0.037 \quad 0] \qquad \eta^T = [0.011 \quad 0.1]$$

$$\Phi^{(1)} = \begin{bmatrix} 0.867 & 0.037 & 0 \\ 0.011 & 0.967 & 0 \\ 0.1 & 0 & 1 \end{bmatrix} \qquad \Phi^{(2)} = \begin{bmatrix} -0.074 \\ 0.001 \\ 0 \end{bmatrix}$$

$$\Phi^{(3)} = [0 \quad -0.1 \quad 0.024] \qquad \Phi^{(4)} = [0.995]$$

$$L^{(1)} = \begin{bmatrix} 0.127 & -2.031 \\ -0.206 & 0.133 \\ 0 & 0 \end{bmatrix} \qquad L^{(2)} = [0.004 \quad 0]$$

$$\mu = 0 \qquad M = [-0.1 \quad 0.024]$$

For the present simulation the weighting matrices and scalars are taken as $Q^* = R^* = I$; $q^* = r^* = 1$; and the computation round-off error parameter $\xi = 0.003$. The computer program was constructed in two principal segments. One segment for testing the operational modes of the three sensors (single-axis roll rate gyro, single-axis yaw rate gyro, and a vertical gyro) by the sensor failure criteria. The other segment for isolating the faulty sensor. The observation subprogram was constructed to indicate a malfunctioning roll rate gyro that starts at step $k = 2$ and the incorrect roll rate signal becomes distorted by 50% of its true value at step $k = 3$. The erroneous signal was detected by the sensor failure criteria at step $k = 3$ and the faulty sensor isolation subprogram started to ignore the signal coming from the failed sensor, rearranged the matrices in the proper forms as indicated in the analysis and reinitialized the sensor failure criteria subprogram which started testing the remaining two sensors (yaw rate gyro and the vertical gyro). Figure 2 shows the results obtained from the computer program regarding the roll rate. The other states estimates (yaw rate, roll angle, and sideslip angle) were not affected at all.

Conclusion

A new technique for sensor failure detection and isolation has been introduced. An adaptive control approach has been used to derive criteria for checking the sensors' operational modes (failure/operational). The detected malfunctioning sensor is automatically isolated (neglecting its output signal), while the information obtained from sensors in the operational mode are used to continue the control process. The presented example demonstrates the practicality and reliability of the introduced algorithm.

References

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